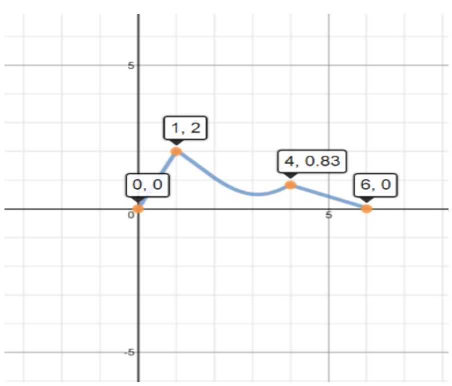
Transformations in Function Notation

Most functions stem from a parent function; for example, $f(x) = x^2 + 7x + 12$ comes from the parent function $g(x) = x^2$. Parent functions and other functions in that same family share many common traits or attributes. Equation can be written in three forms: intercept form, standard form and vertex form. Intercept form, example $b(x) = a(x-b)(x-c)(x-d)$, is great for identifying x-intercepts. Standard form like the equation $f(x)$ above is good for identifying the y-intercept. The best form for graphing is vertex form, or transformation form. If you know the parent function and understand how each parameter effect the function, graphing will be very simple.

Type of Function	*Vertex Form*	*Transformation Form*
Linear	$f(x) = a(x-h) + k$	$f(x) = a(bx-c) + d$
Quadratic	$f(x) = a(x-h)^2 + k$	$f(x) = a(bx-c)^2 + d$
Cubic	$f(x) = a(x-h)^3 + k$	$f(x) = a(bx-c)^3 + d$
Square Root	$f(x) = a\sqrt{x-h} + k$	$f(x) = a\sqrt{bx-c} + d$
Rational	$f(x) = \frac{a}{x-h} + k$	$f(x) = \frac{a}{bx-c} + d$
Absolute Value	$f(x) = a\lvert x-h\rvert + k$	$f(x) = a\lvert bx-c\rvert + d$

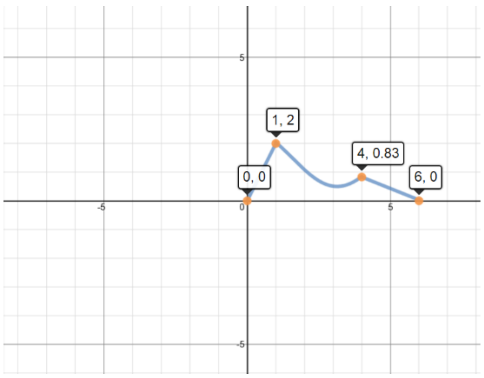
Notice the similarity between all the vertex form and transformation form. All contain the parameters $a, b, c, and\ d$. Each parameter effect the behavior of the graph. For the sack of simplicity just consider a general function, $\boldsymbol{g(x) = af(bx-c) + d}$, where $f(x)$ is the original function. Each parameter will be explored separately from the others.



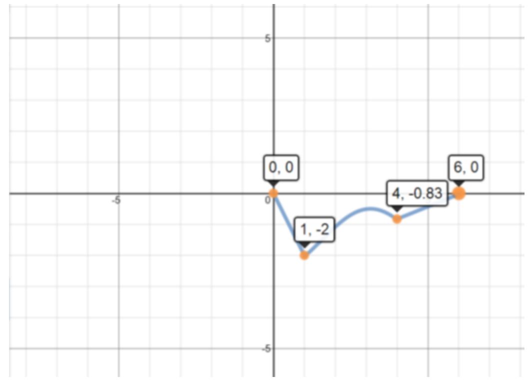
Function: $f(x)$

Consider this the original function

" $\pm a$ " -Reflection over the x-axis



$g(x) = +f(x)$

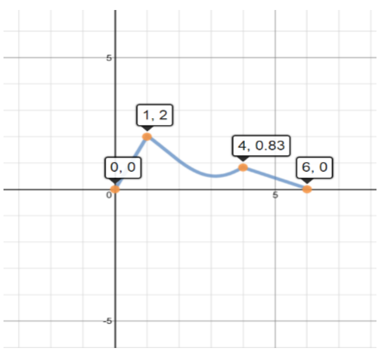


$h(x) = -f(x)$

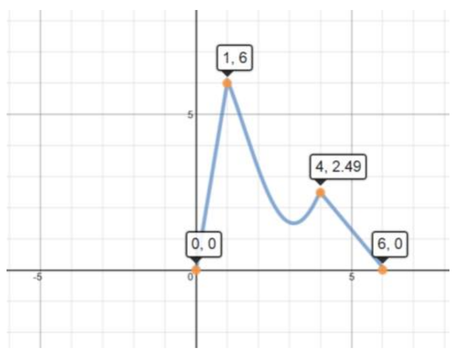
Note:

The negative produces a reflection over the x-axis; this reflection causes the opposite y-value with the same x-value of the original function. EX: $(1,2) \rightarrow (1, -(2)) \rightarrow (1, -2)$

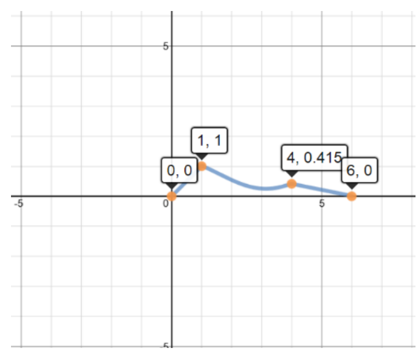
"$|a|$"- Vertical Stretch or Compression



$f(x)$



$g(x) = 3f(x)$



$h(x) = \frac{1}{2}f(x)$

Note:

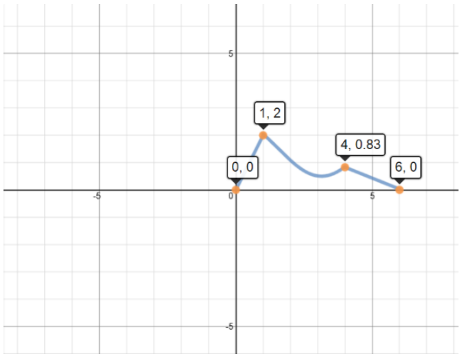
$g(x)$ represents a vertical stretch; $3f(x)$ can also be thought of as $3y$ so you can take the y-values of $f(x)$ and multiply it by 3. The x-value of the point will remain the same.
EX: $(1,2) \rightarrow (1, 3(2)) \rightarrow (1,6)$.

$h(x)$ represents a vertical compression; again $\frac{1}{2}f(x) = \frac{1}{2}y$ while the x-value remains the same.

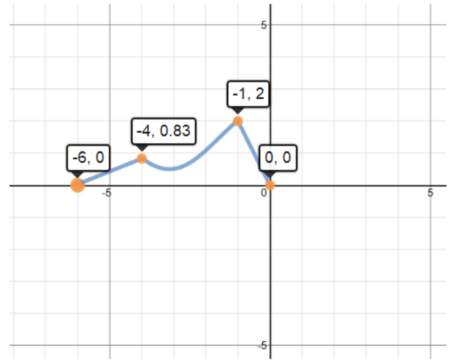
EX: $(1,2) \rightarrow \left(1, \frac{1}{2}(2)\right) \rightarrow (1,1)$

Simple Rule: $If\ a > 1\ it\ is\ a\ vertical\ stretch\ and\ if\ 0 < a < 1\ it\ is\ a\ vertical\ compression$

" $\pm b$" -Reflection over the y-axis



$g(x) = f(x)$

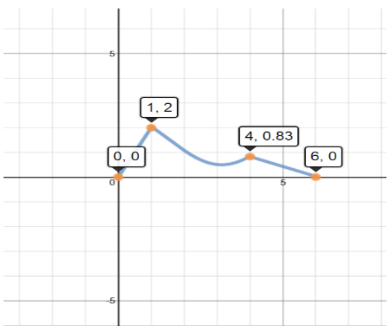


$h(x) = f(-x)$

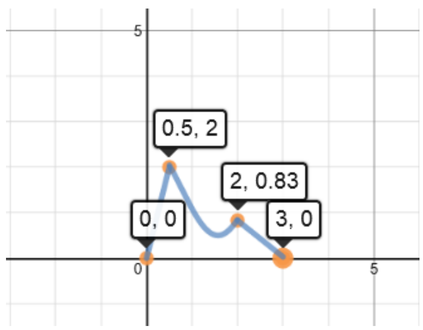
Note:

The negative produces a reflection over the y-axis; this reflection causes the opposite x-value with the same y-value of the original function. EX: $(1,2) \rightarrow (-(1),\ 2) \rightarrow (-1, 2)$

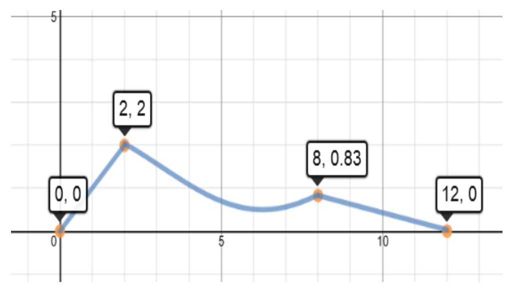
"$|b|$"- Horizontal Stretch or Compression



$f(x)$



$g(x) = f(2x)$



$h(x) = f\left(\frac{1}{2}x\right)$

Note:

$g(x)$ represents a horizontal compression; $f(2x)$ which implies that you x-values on $f(x)$ and divide it by $\frac{1}{2}$. The y-value of the point will remain the same. EX: $(1,2) \rightarrow \left(\left(\frac{1}{2}\right)1, 2\right) \rightarrow (\frac{1}{2}, 2)$.

$h(x)$ represents a horizontal stretch; again $f\left(\frac{1}{2}x\right) = 2(x - value)$ while the y-value remains the same. EX: $(2(1),2) \rightarrow (2(1), 2) \rightarrow (2,2)$

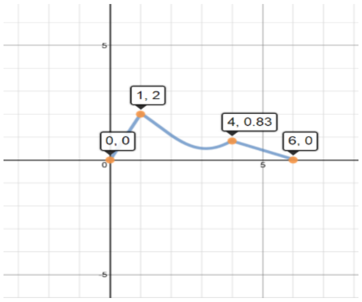
Simple Rule: $If\ b > 1\ it\ is\ a\ horizontal\ compression\ and\ if\ 0 < b < 1\ it\ is\ a\ horizontal\ stretch$

Warning: For many graphs a horizontal compression appears as a vertical stretch and also a horizontal stretch appears as a vertical compression.

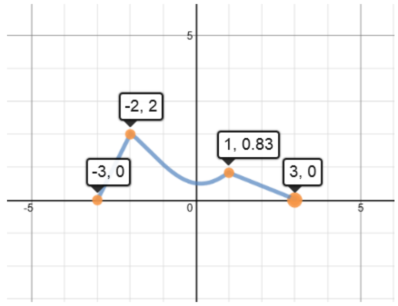
Example: Consider $f(x) = 4x^2$, a quadratic, which has been vertically stretched by a factor of 4.

$g(x) = (2x)^2$, also a quadratic, which has a horizontal compression. Algebraic look closer $g(x) = (2x)^2 = (2x)(2x) = 4x^2 = f(x)\ \ so\ g(x) = f(x)\ so\ a\ vertical\ stretch\ can\ by\ a\ horizontal\ compression\ as\ well.$

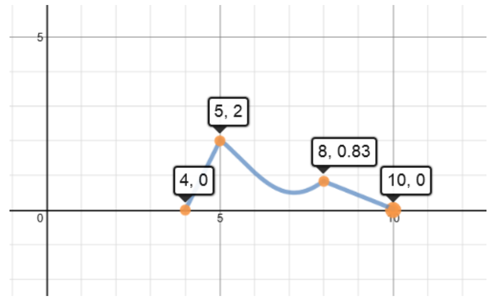
"c"- Horizontal Shift (translation left or right)



$\boldsymbol{f(x)}$



$\boldsymbol{g(x) = f(x + 3)}$

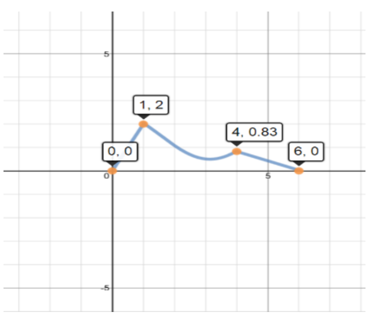


$\boldsymbol{h(x) = f(x - 4)}$

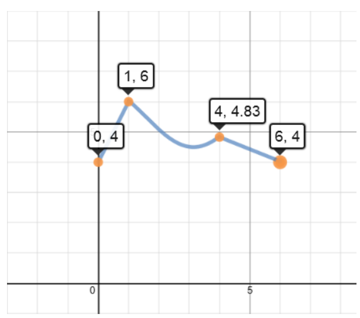
Note:

This transformation works a little opposite to what one may think but it is because the general form defines it as $f(x - c)$. The negative inside means "opposite" so like in $g(x)$ one might first assume that $f(x + 3)$ would move the graph 3 units right; however, the graph was moved 3 units left. This is because to change the sign $c = -3 \rightarrow f\big(x - (-3)\big) = f(x + 3)$ and since c is negative that gets us movement to the left. So notice that $f(x - 4)$ moved the graph to the right since c is 4 which is a positive. $EX\ of\ g(x)\text{:}\ (1,2) \rightarrow (1 - 3,\ 2) \rightarrow (-2, 2)$

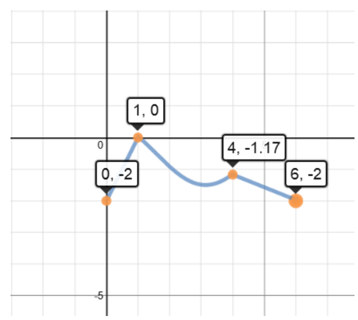
"d"- Vertical Shift (translation up or down)



$f(x)$



$g(x) = f(x) + 4$



$h(x) = f(x) - 2$

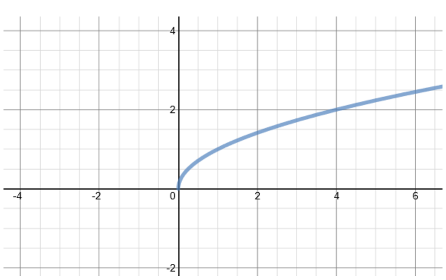
Note:

Do not worry here, the function will behave like it is expected to.

$$g(x) = f(x) + 4\ moved\ the\ graph\ up\ while\ h(x) = f(x) - 2\ moved\ the\ graph\ down.$$

$$EX\ of\ g(x)\text{:}\ (1,2) \rightarrow (1, 2 + 4) \rightarrow (1, 6)$$

Example: $parent\ function\ f(x) = \sqrt{x} \quad \rightarrow \quad p(x) = 2\sqrt{x - 1} + 5$



Transformations:

Vertical stretch

Right 1 unit

5 units up