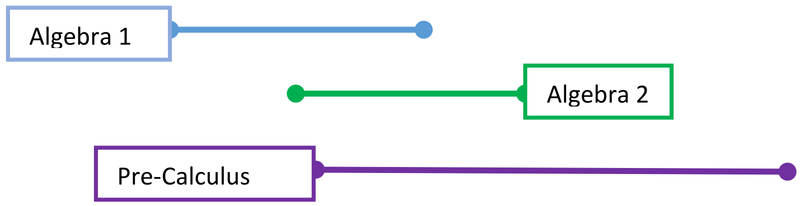
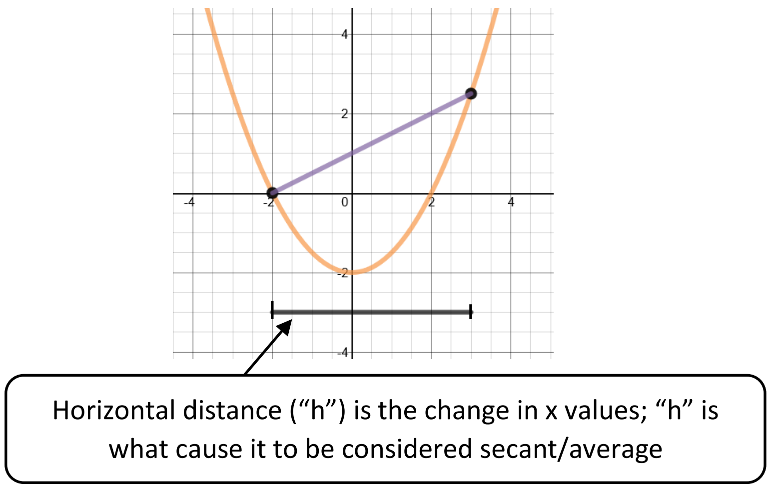
Slope of the Secant and Tangent

Slope, better known as average rate of change, measures the change in one quantity as a ratio in respect to the change in another quantity. Average rate of change has many representations:

$$Average\ rate\ of\ change = slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$



The ***secant slope*** is an average rate of change because the slope based on two point separated by horizontal distance.



A 12 ounce can of soda is punctured forming a small hole. The volume (in ounces) of the can represent the remaining volume in the can after t seconds.

t (seconds)	2	4	6	8	10
V (ounces)	11	8	4	1	0

What is the average rate of change on the interval [4,8]?

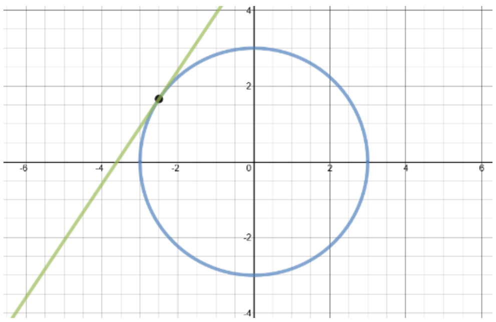
$$m_{sec} = \frac{v(8) - v(4)}{8 - 4} = \frac{1 - 8}{8 - 4} = \frac{-7}{4}\frac{oz.}{sec}$$

Thus, the volume is decreasing at an average rate of $\frac{7}{4}$ ounces every second.

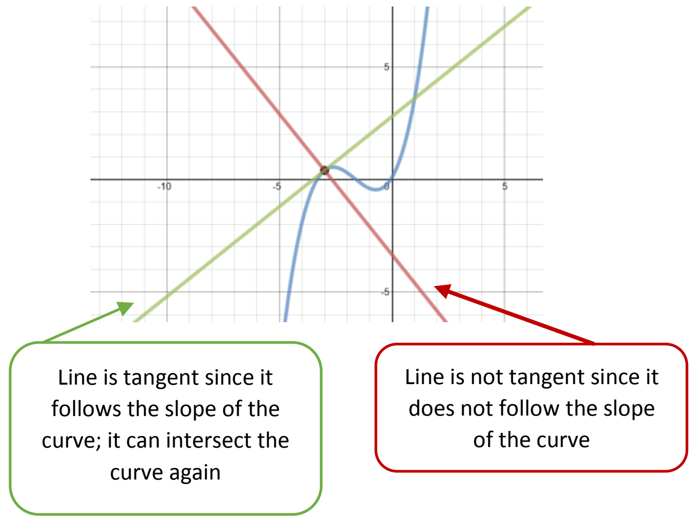
- What if we wanted to know the rate the soda is leaking out at time 5?

The question now shifts from being an average rate of change (two points) to an instantaneous rate of change (a single point). Just as secant refers to average rate of change, a tangent refers to the instantaneous rate of change.

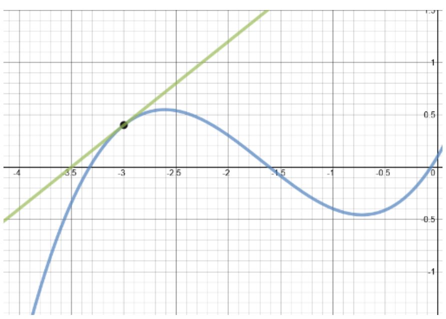
The ***slope of the tangent*** is an instantaneous rate of change $\left[\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{dy}{dx}\right]$ because it is slope based on a single point and the slope of the tangent at a point on the curve will follow the slope of the curve.



Line is tangent to the circle since it follows the slope of the curve and only intersects at a single point.

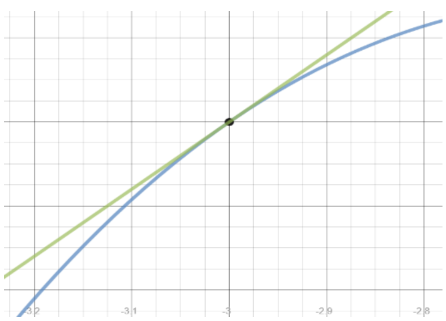


A closer look at a tangent line and its curve: LOCAL LINEARITY

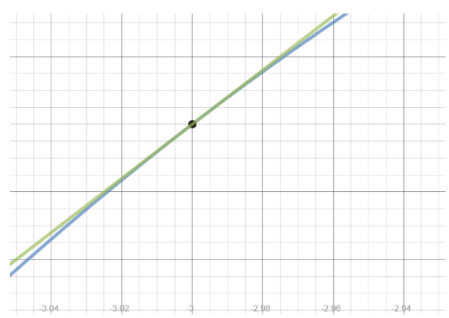


Standard view of tangent and curve

Zooming in on point of tangency



Further zooming in



Zoomed in to where the curve and the tangent line appears to be one

So referring to the question above, how do you find the slope at a particular point? Throughout this course we will develop more accurate tools to find instantaneous rate of change. However, we can approximate the slope at a point using the secant slope, m_{sec}.

A 12 ounce can of soda is punctured forming a small hole. The volume (in ounces) of the can represent the remaining volume in the can after t seconds.

t (seconds)	2	4	6	8	10
V (ounces)	11	8	4	1	0

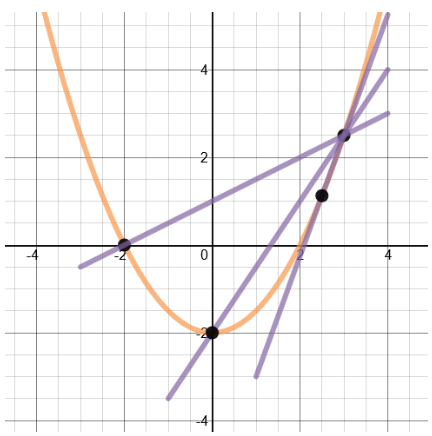
What is the rate at which the soda is leaking out at 5 seconds?

- The closes values to 5 that are stated, since we are confined to the value provided in the table, is time 4 and time 6. Thus,

$$m_{sec} = \frac{v(6) - v(4)}{6 - 4} = \frac{4 - 8}{8 - 4} = \frac{-4}{4} = -1\frac{oz.}{sec}$$

Thus, the volume is decreasing approximately at an average rate of 1 ounces every second at 5 seconds.

A table is very limiting in the ability to best approximate the rate of change at a given point; however, when given an equation we can choose two points (so that we can still using math that we know) that are significantly close to the point of interest.



Notice as points are taken closer and closer to $x = 3$ which will minimize the horizontal distance between the two points; the slope of the secant line starts to become linear with the curve. This will provide us with a good approximation of the instantaneous rate of change at a point.

Example: Find the instantaneous rate of change on the curve $f(x) = e^x - 3$ at $x = 2$.

☆ Note: must find $(2, f(2))$ *to complete table so* $(2, 4.389)$

$\boldsymbol{x}$	1	1.5	1.9	1.99	1.999
$\boldsymbol{f(x)}$	−.282	1.482	3.686	4.316	4.382
$\boldsymbol{\Delta y}$	−4.671	−2.907	−.703	−.074	−.007
$\boldsymbol{\Delta x}$	−1	−.5	−.1	−.01	−.001
$\boldsymbol{\frac{\Delta y}{\Delta x}}$	4.671	5.815	7.032	7.352	7.385

Appears that the slope is approaching a specific value as we approach 2 → $m_{sec} \approx 7.385$

The instantaneous rate of change at $x = 2$ *on* $f(x)$ *is approximately* 7.385.

Once you approximate the instantaneous rate of change at a given point we can then write an equation for a line tangent at that point.

Best way to do this is using the point-slope form: $y - y_1 = m(x - x_1)$ or $y = m(x - x_1) + y_1$

Using the example above we approximated the slope of the tangent to be 7.385 and the point of tangency was $(2, 4.389)$.