Transformations in Function Notation

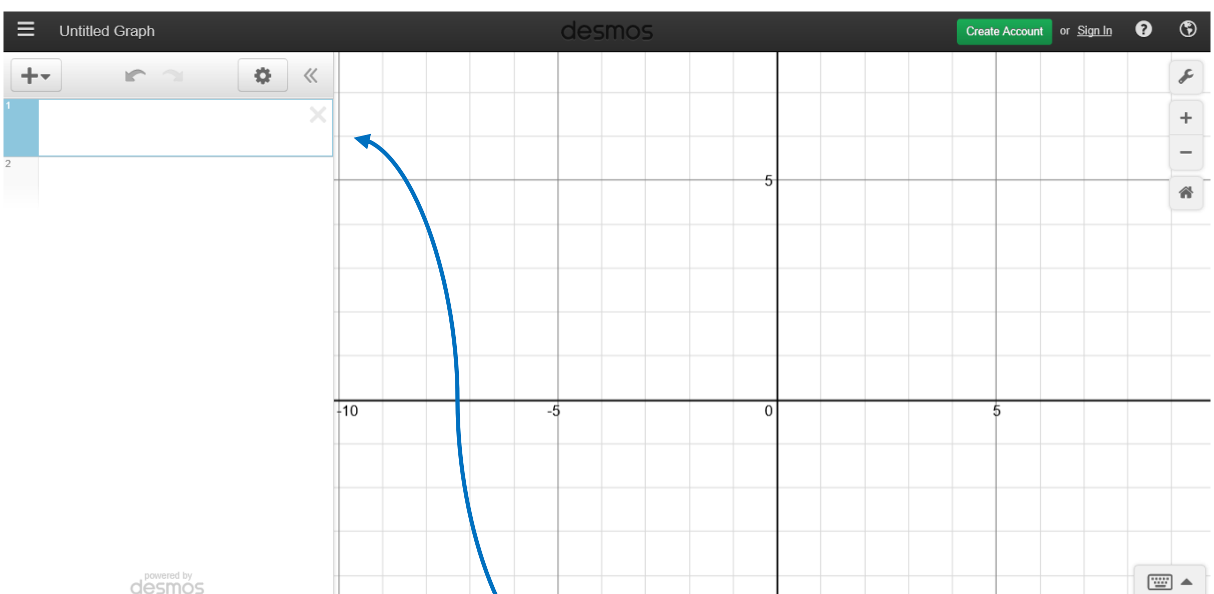
Most functions stem from a parent function; for example, $f(x) = x^2 + 7x + 12$ comes from the parent function $g(x) = x^2$. Parent functions and other functions in that same family share many common traits or attributes. Equation can be written in three forms: intercept form, standard form and vertex form. Intercept form, example $b(x) = a(x - b)(x - c)(x - d)$, is great for identifying x-intercepts. Standard form like the equation $f(x)$ above is good for identifying the y-intercept. The best form for graphing is vertex form, or transformation form. If you know the parent function and understand how each parameter effect the function, graphing will be very simple.

Type of Function	*Vertex Form*	*Transformation Form*
Linear	$f(x) = a(x - h) + k$	$f(x) = a(bx - c) + d$
Quadratic	$f(x) = a(x - h)^2 + k$	$f(x) = a(bx - c)^2 + d$
Cubic	$f(x) = a(x - h)^3 + k$	$f(x) = a(bx - c)^3 + d$

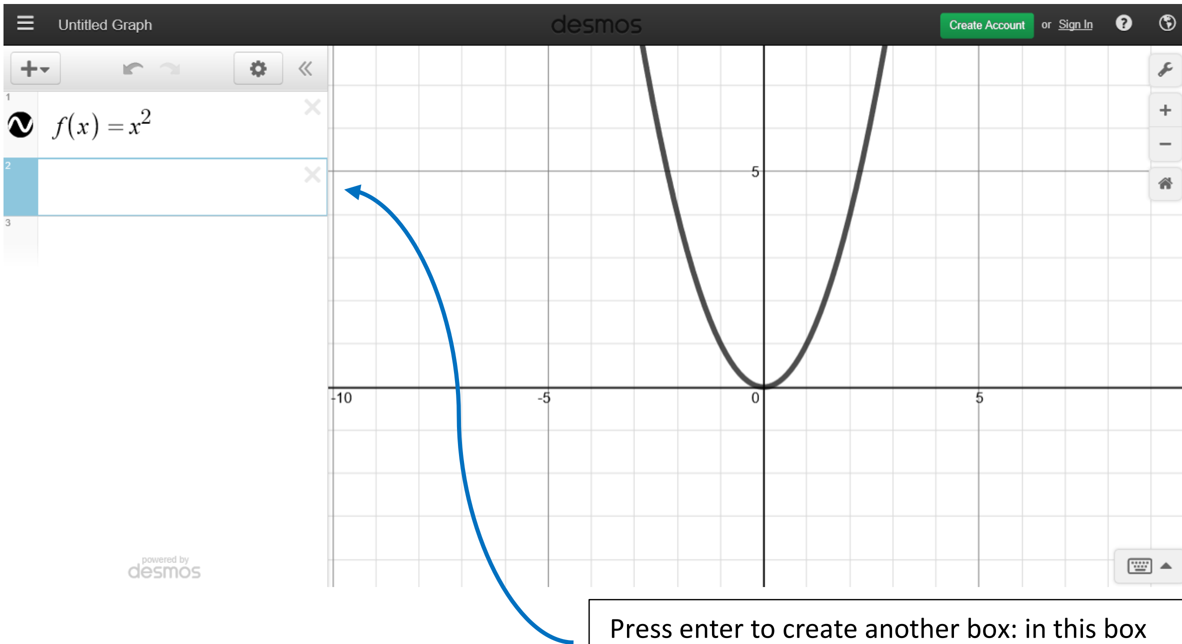
Notice the similarity between all the vertex form and transformation form. The real difference between vertex form and transformation form is the parameters b which we will kind of ignore for this lesson. Each parameter effect the behavior of the graph. If you understand the effect that each parameter has graph and writing an equation for a function is much easy in vertex form then standard.

In the last lesson you focused on quadratics in standard form. Standard form requires a lot more work and algebra, vertex does not. Recall: The vertex is the turning point in the quadratic and this point is key to identifying other key characteristics.

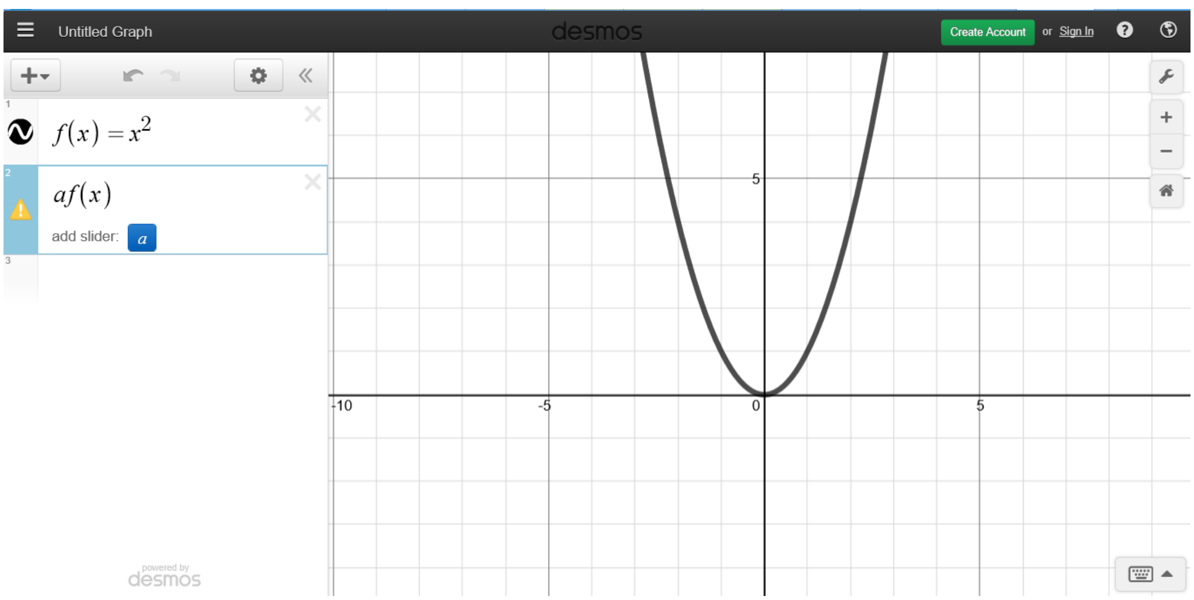
LET EXPLORE goes to desmos.com and click launch calculator.



Type equations here: Type the equation $f(x) = x^2$ which is the parent equation



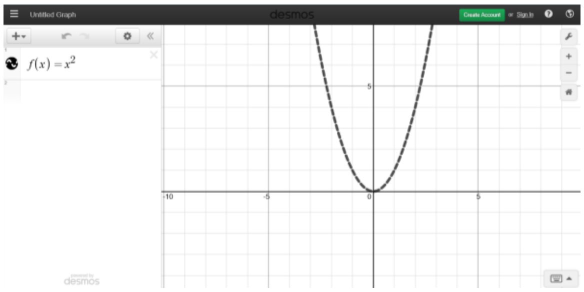
Press enter to create another box: in this box type $af(x)$: then click “a” in bottom of the box to turn it into a slider then press play



You can create another box and type $f(x-h)$ and another box and type $f(x)+k$; then turn both “h” and “k” into sliders and explore the effect that each parameter have on the parent function, $f(x)=x^2$. The next page will summarize the effects of the parameters.

Vertex Form of Quadratics

In order to understand the effect that each parameter ("a", "h", and "k") we must first graph the parent function, $f(x) = x^2$. All parent function go through the origin thus we can rewrite the parent function in vertex form as $f(x) = 1(x-0)^2 + 0$. Through the exploration we will soon make sense why there are 1 and 0s. Lets make some key observations about the parent function which will be represented as a dotted line in all examples.



Note: Parent Function

Vertex: (0,0)

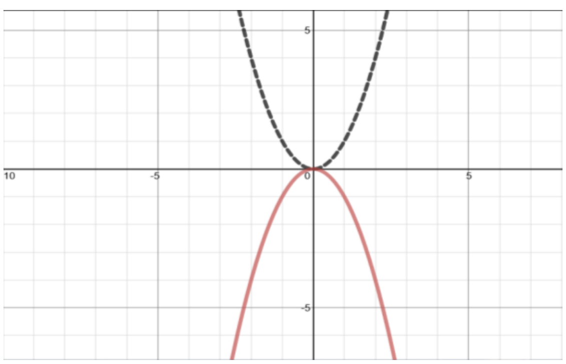
Axis of Symmetry: x=0

Concavity: Concave Up

$$f(x) = a(x-h)^2 + k$$

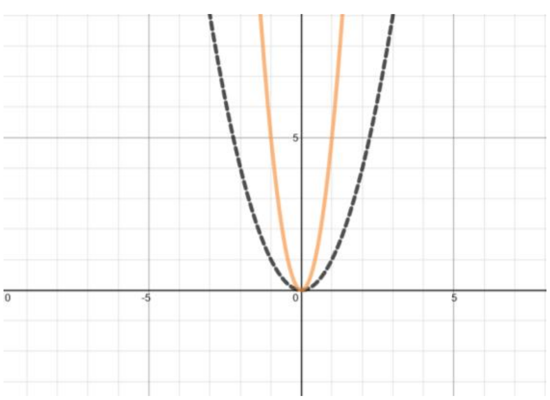
The parameter "a" can have to effects on the parent function:

1. It can cause a reflection over the x-axis.

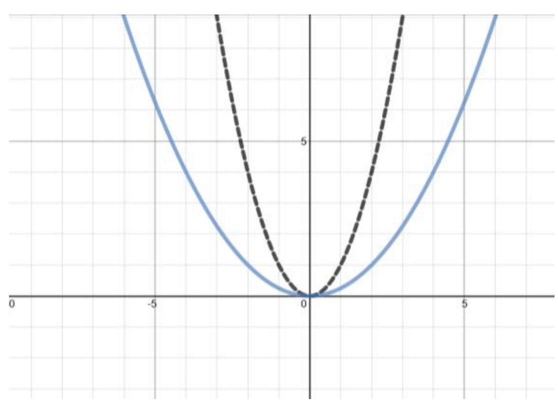


- If "a" is a positive value then the function is concave up.
- If "a" is a negative value then the function is concave down.

2. It can cause a vertical stretch or vertical compression



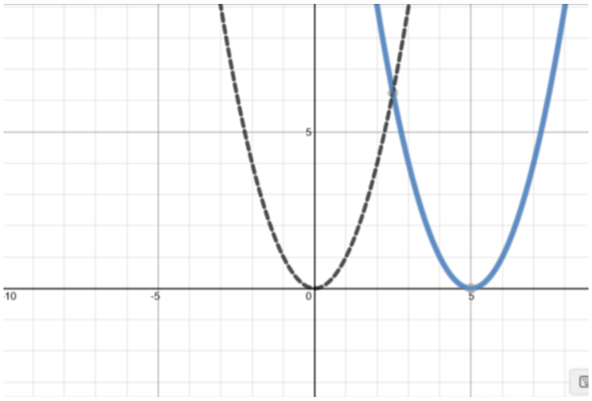
If $|a| > 1$ it is a vertical stretch

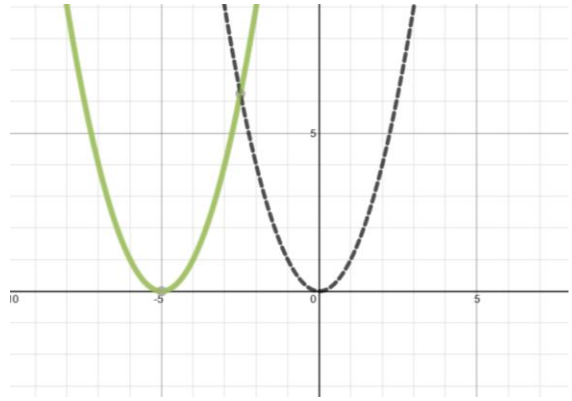


If $0 < |a| < 1$ it is a vertical compression

$$f(x) = a(x - h)^2 + k$$

The parameter "h" simply shifts the graph left or right. But be careful "h" does the opposite of what you think because of the negative that is in front of it in the general equation.





If $f(x - h)$ then the graph is shifted to the right

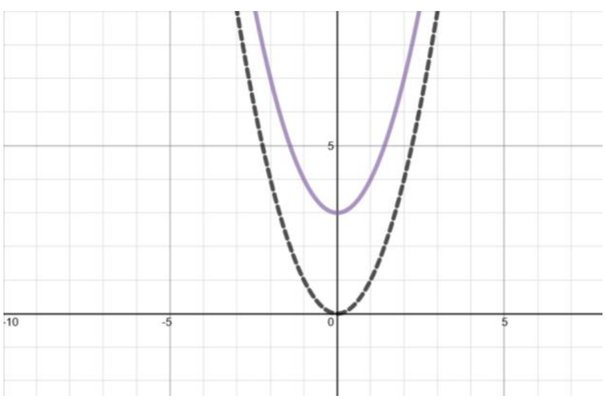
Example: $f(x) = (x - 5)^2$ *as shown above*

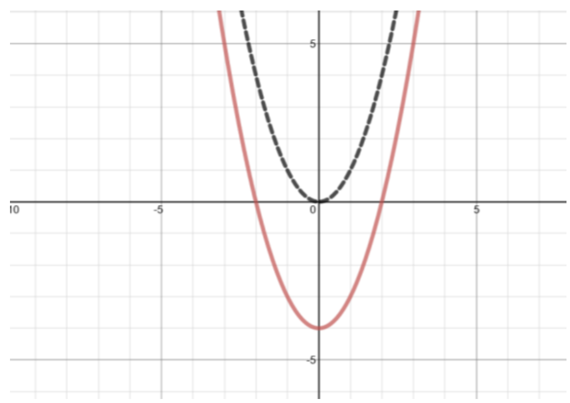
If $f(x + h)$ then the graph is shifted to the left

Example: $f(x) = (x + 5)^2$ *as shown above*

$$f(x) = a(x - h)^2 + k$$

The parameter "k" behaves normal. It moves the graph up and down.





If $f(x) + k$ then the graph is shifted to the up

Example: $f(x) = x^2 + 3$ *as shown above*

If $f(x) - k$ then the graph is shifted to the down

Example: $f(x) = x^2 - 4$ *as shown above*

Note: These transformation move the vertex away from the origin. So the key is if you have an equation it is easy to identify the vertex of the quadratic since (h, k) *is the vertex we can just use the equation* $f(x) = a(x - h)^2 + k$ *by looking at it*.

Examples on next page.

Example: Graph $f(x) = \frac{1}{2}(x+2)^2 - 6$

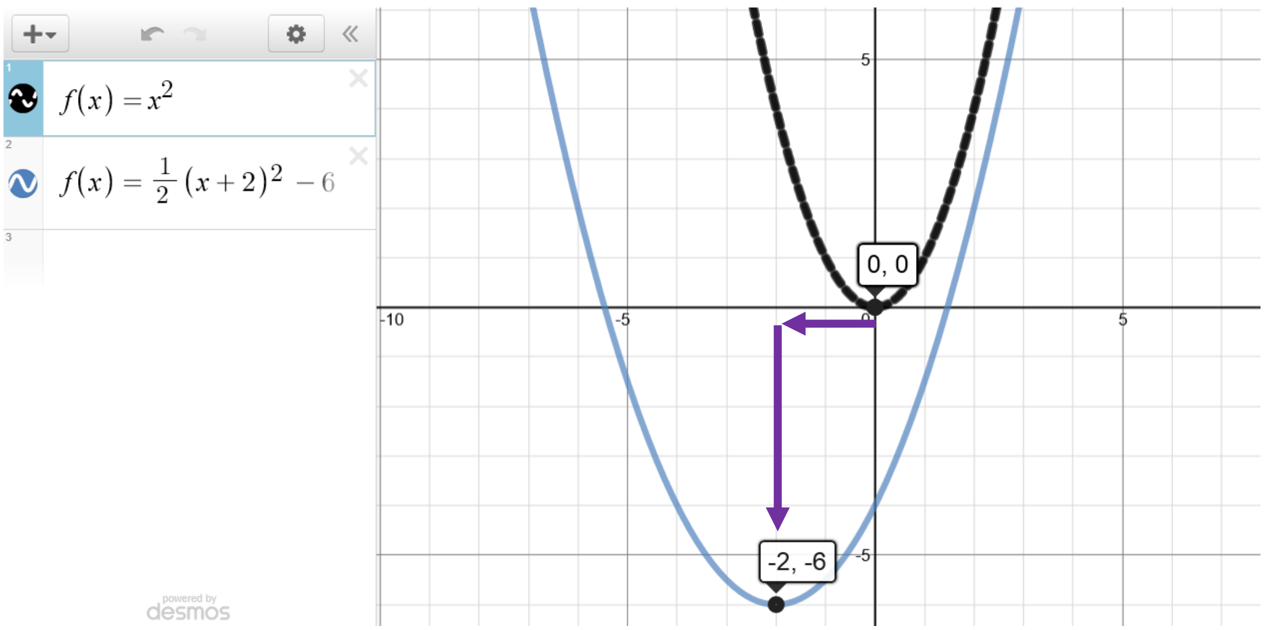
Vertex: $(-2, -6)$ *remember "h" is opposite

Axis of Symmetry: $x = -2$

Concavity: Concave Up because $a = +\frac{1}{2}$

Transformations:

- Vertical Compression
- Shifted to the left 2
- Shifted down 6



From here you can also talk about if the vertex is a maximum or minimum. You can indicate the intervals of increasing and decreasing. Vertex makes graphing easy without the aid of a calculator; it will also make equation writing easier in the next lesson.