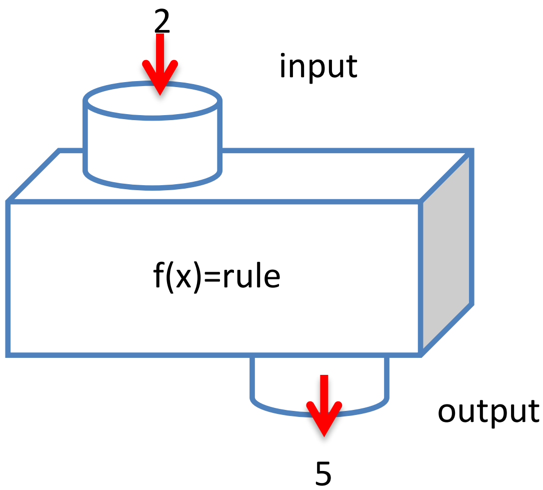
Operations on Functions

Need to Know: Introducing the function machine. The function machine will be very helpful in understanding how operations on functions work. Remember that to be a function that for every unique input (or x-value) the function produces a unique output (y-value).



Given the points $\{(2,5),(-1,-3),(0,4)\}$

Each input produces a unique output→this is a function

Given the points $\{(2,7),(3,-4),(2,-2)\}$

Notice that 2 will produce the outputs 7 and -2→ so this is not a function.

Just as numbers can be added, subtracted, multiplied, and divided the same can be done with equations.

Notations: Order of function ***does matter***

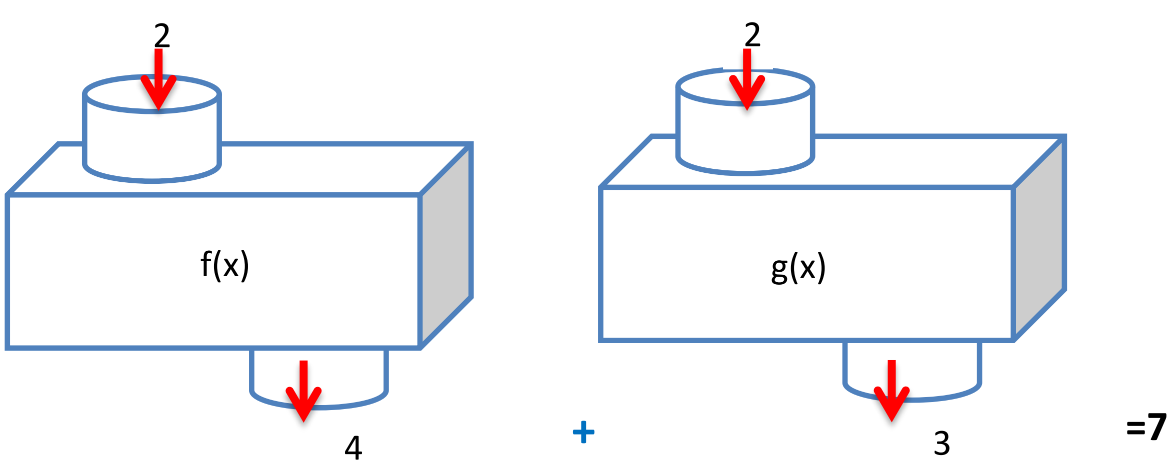
$(f+g)(x)=f(x)+g(x)$ $\qquad$ $(g+f)(x)=g(x)+f(x)$

$(f-g)(x)=f(x)-g(x)$ $\qquad$ $(g-f)(x)=g(x)-f(x)$

$(fg)(x)=f(x)g(x)$ $\qquad$ $(gf)(x)=g(x)f(x)$

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$, where $g(x)\neq 0$ $\qquad$ $\left(\frac{g}{f}\right)(x)=\frac{g(x)}{f(x)}$, *where* $f(x)\neq 0$

VISUAL EXAMPLE: $f(x)=2x$ *and* $g(x)=x+1$ *determine* $(f+g)(2)=f(2)+g(2)$



ALGEBRAIC REPRESENTATION

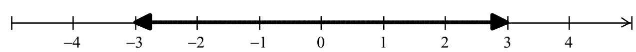
Given that functions are being added, subtracted, multiplied, or divided one does have to keep in mind the domain. Especially the domain, when division is done needs a lot more attention.

Guided Example: Given $f(x) = 2x + 1$ *and* $g(x) = x^2 + x$

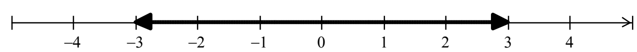
$(\boldsymbol{f} + \boldsymbol{g})(\boldsymbol{x})$	Separation of terms	$(\boldsymbol{g} + \boldsymbol{f})(\boldsymbol{x})$
$= f(x) + g(x)$	Substitution	$= g(x) + f(x)$
$= (2x + 1) + (x^2 + x)$	Drop parenthesis if there is not any distribution or exponential expansions	$= (x^2 + x) + (2x + 1)$
$= 2x + 1 + x^2 + x$		$= x^2 + x + 2x + 1$
$= x^2 + 3x + 1$	Combine like terms	$= x^2 + 3x + 1$

To find the domain you must consider the domain of each function.

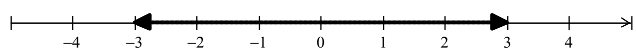
Domain: $f(x): x \in \mathbb{R}$ $(-\infty, \infty)$



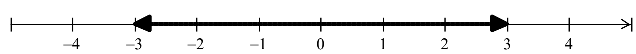
$g(x): x \in \mathbb{R}$ $(-\infty, \infty)$



$(\boldsymbol{f} + \boldsymbol{g})(\boldsymbol{x}): x \in \mathbb{R}$ $(-\infty, \infty)$



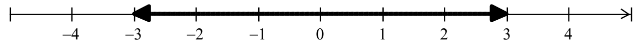
$(\boldsymbol{g} + \boldsymbol{f})(\boldsymbol{x}): x \in \mathbb{R}$ $(-\infty, \infty)$



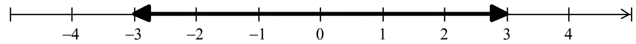
$(\boldsymbol{f} - \boldsymbol{g})(\boldsymbol{x})$	Separation of terms	$(\boldsymbol{g} - \boldsymbol{f})(\boldsymbol{x})$
$= f(x) - g(x)$	Substitution	$= g(x) - f(x)$
$= (2x + 1) - (x^2 + x)$	Negative sign needs to be distributed across the second term	$= (x^2 + x) - (2x + 1)$
		$= x^2 + x - 2x - 1$
$= 2x + 1 - x^2 - x$	Combine like terms	$= x^2 - x - 1$
$= -x^2 + x + 1$		

Notice that within subtraction that order does matter just as $3 - 5$ is not the same as $5 - 3$

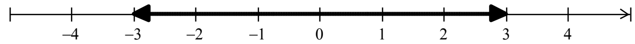
Domain: $f(x): x \in \mathbb{R}$ $(-\infty, \infty)$



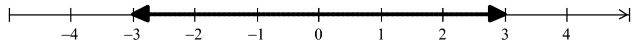
$g(x): x \in \mathbb{R}$ $(-\infty, \infty)$



$(\boldsymbol{f}-\boldsymbol{g})(\boldsymbol{x}): x \in \mathbb{R}$ $(-\infty, \infty)$

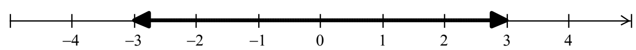


$(\boldsymbol{g}-\boldsymbol{f})(\boldsymbol{x}): x \in \mathbb{R}$ $(-\infty, \infty)$

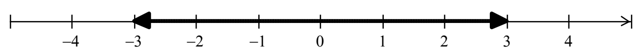


$(\boldsymbol{fg})(\boldsymbol{x})$	Separation of terms	$(\boldsymbol{gf})(\boldsymbol{x})$
$= f(x)g(x)$	Substitution	$= g(x)f(x)$
$= (2x+1)(x^2+x)$	Since this is multiplication use distribution (or FOIL)	$= (x^2+x)(2x+1)$
$= 2x^3 + 2x^2 + 1x^2 + 1x$	Combine like term	$= 2x^3 + 1x^2 + 2x^2 + 1x$
$= 2x^3 + 3x^2 + x$		$= 2x^3 + 3x^2 + x$

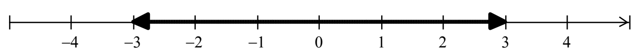
Domain: $f(x): x \in \mathbb{R}$ $(-\infty, \infty)$



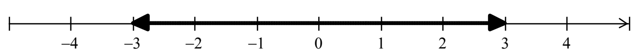
$g(x): x \in \mathbb{R}$ $(-\infty, \infty)$



$(\boldsymbol{fg})(\boldsymbol{x}): x \in \mathbb{R}$ $(-\infty, \infty)$



$(\boldsymbol{gf})(\boldsymbol{x}): x \in \mathbb{R}$ $(-\infty, \infty)$



$\left(\frac{f}{g}\right)(x)$

$= \frac{f(x)}{g(x)}$

$= \frac{2x+1}{x^2+x}$

$= \frac{2x+1}{x(x+1)}$

Separation of terms

Substitution

Make sure to factor top and bottom this is needed for domain and range

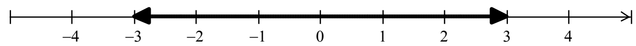
$\left(\frac{g}{f}\right)(x)$

$= \frac{g(x)}{f(x)}$

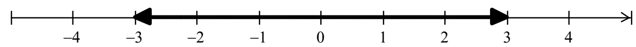
$= \frac{x^2+x}{2x+1}$

$= \frac{x(x+1)}{2x+1}$

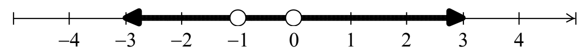
Domain: $f(x): x \in \mathbb{R}$ $(-\infty, \infty)$



$g(x): x \in \mathbb{R}$ $(-\infty, \infty)$

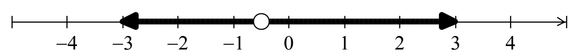


$\left(\frac{f}{g}\right)(x): x \in \mathbb{R} \neq -1,0 \quad -\infty, -1) \cup (-1, 0) \cup (0, \infty)$



The division requires $x = -1$ *and* 0 to be removed

$\left(\frac{g}{f}\right)(x): x \in \mathbb{R} \neq -\frac{1}{2} \quad (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

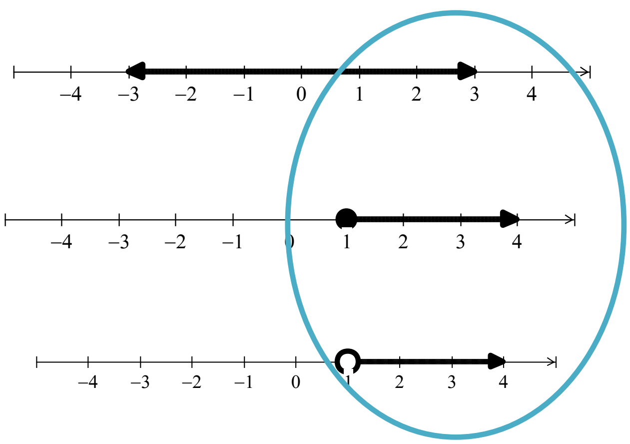


Better Example

$f(x) = -x + 3$ $D: (-\infty, \infty)$

$g(x) = \sqrt{x-1}$ $D: [1, \infty)$

$\left(\frac{f}{g}\right)(x) = \frac{-x+3}{\sqrt{x-1}}$ $D: (1, \infty)$



Notice that the domain is a compromise between the domain of $f(x)$ *and* $g(x)$, but it also takes into account that there is no division by 0 which happens when $x = 1$.