Graphing Quadratics in Standard Form

What is standard form: Standard form for a quadratic is $f(x) = ax^2 + bx + c$. Standard form is not the friendliest form to work with; however, it is the most common form used in math. There are many key characteristics of a quadratic function that we can identify:

- Vertex → which is also the absolute maximum or absolute minimum
- Axis of Symmetry
- Y-intercept
- Intervals of increasing/decreasing
- Concavity
- X-intercepts→this will be covered later because alone it is a big and important characteristic

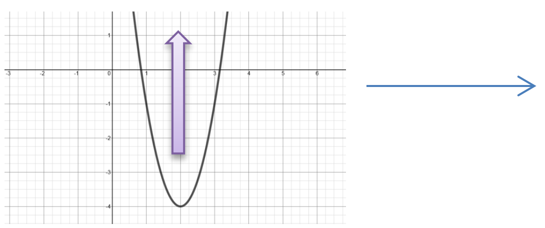
Y-intercept: All y-intercepts as a coordinate point look like $(0, y)$ since it is where the graph crosses the x-axis. To algebraically find the y-intercept you must simply plug in 0 for x and solve for y.

Example: Consider the quadratic, $g(x) = -2x^2 + 5x - 6$, if we plug in $x = 0$ we get

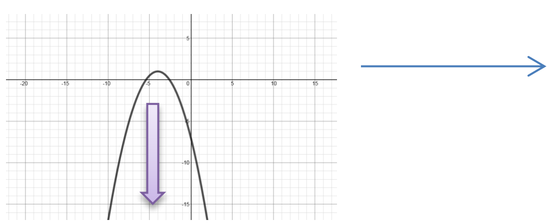
$g(0) = -2(0)^2 + 5(0) - 6 = 0 + 0 - 6 = -6$; therefore, the y-intercept is $(0, -6)$.

Notice that the zeros make all those terms with an x cancel out. This is always true so we can say that the y-intercept is $(0, c)$ when the function is in standard form, $ax^2 + bx + c$.

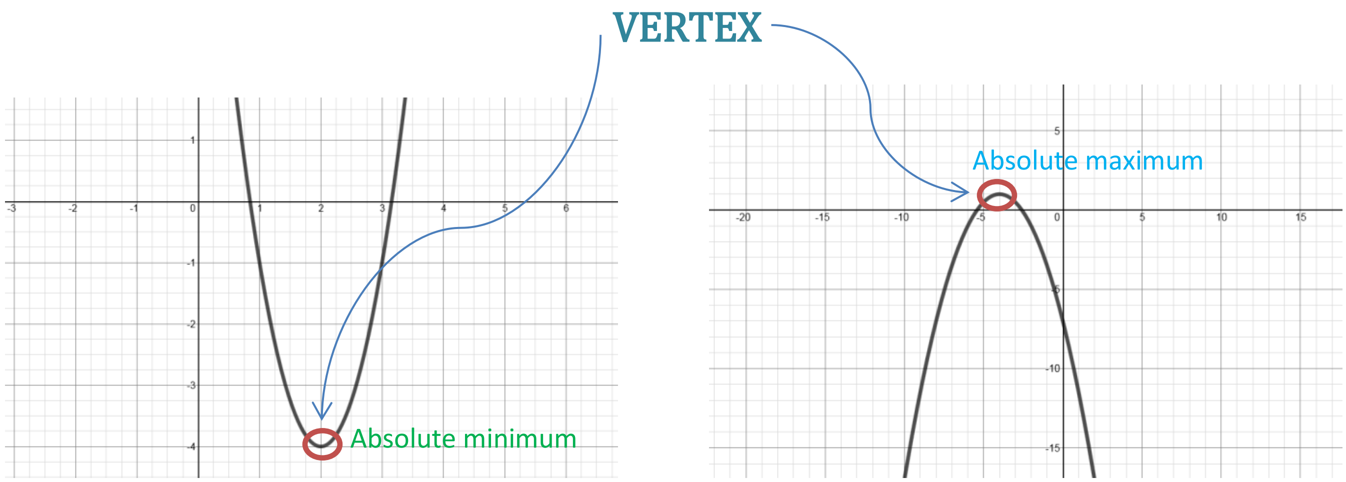
Concavity: In simple terms, concavity is the direction that the function opens.



This graph opens up which says that the leading coefficient (or "a" in standard form) is positive. This is called CONCAVE UP



This graph opens down which says that the leading coefficient (or "a" in standard form) is negative. This is called CONCAVE DOWN



The vertex is a key characteristic that needs to be identified in order to determine other characteristics. The vertex is where the quadratic turns (or changes from increasing to decreasing); it is also to be considered the absolute minimum (if it is the lowest point) or absolute maximum (if it is the high point). To be an absolute minimum or maximum it must be a concrete value; it cannot be infinity.

To find the vertex algebraically of a function in standard form use $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

Let's look at an example.

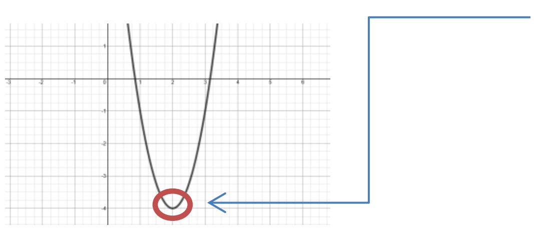
Consider the function, $f(x) = 3x^2 - 12x + 8 \rightarrow a = 3,\ b = -12, and\ c = 8$

$$-\frac{b}{2a} = -\frac{-12}{2(3)} = \frac{12}{6} = 2 \rightarrow \quad this\ means\ that\ x = 2\ so\ 2\ is\ the\ x\ coordinate\ of\ the\ vertex$$

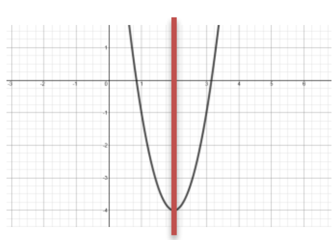
Now plug in the value found, $x = 2$, into the function to find the y-value.

$$f(2) = 3(2)^2 - 12(2) + 8 = 12 - 24 + 8 = -4$$

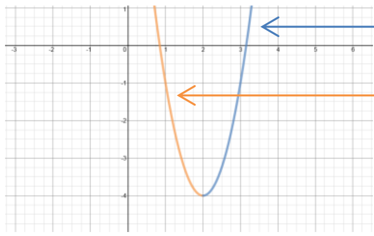
So the vertex of the function is at $(2, -4)$ which can be verified from the graph.



Axis of Symmetry: Quadratics are symmetric and the line of symmetry cuts through the vertex. So for this graph the axis of symmetry is $x = 2$.



Once you have the vertex of the function you can now state on what interval the function is increasing and decreasing.



- The function is increasing when $x > 2$ or $(2, \infty)$.
- The function is decreasing when $x < 2$ or $(-\infty, 2)$.
- Since $x = 2$ is a transition point it is not included

Example Problem: State the following characteristics of the quadratic, $f(x) = -2x^2 + 4x + 3$

- Y-intercept: $f(0) = -2(0)^2 + 4(0) + 3 = 3$ so the y-intercept is at (0,3).
- Concavity: Since the $a = -2$, this quadratic is concave down (it faces down)
- Vertex:
  - $-\frac{b}{2a} = -\frac{4}{2(-2)} = -\frac{4}{-4} = 1$ $so\ the\ x - value\ of\ the\ vertex\ is$ 1
  - $f\left(-\frac{b}{2a}\right) = f(1) = -2(1)^2 + 4(1) + 3 = -2 + 4 + 3 = 5$

    $so\ the\ y - value\ of\ the\ vertex\ is$ 5
    - THE VERTEX is at (1,5)
- Axis of Symmetry: Since the vertex is at (1,5) $the\ axis\ of\ symmetry\ is\ x = 1$.
- Increasing: Since the quadratic is concave down ∩, it is increasing on the interval $(-\infty, 1)$ or when $x < 1$.
- Decreasing: Since the quadratic is concave down ∪, it is decreasing on the interval $(1, \infty)$ or when $x > 1$.

Now with all the information above plot the y-intercept and the vertex. Then using the concavity draw the quadratic. This will be a rough sketch but we don't need sometimes to be exact.