Factoring

What is a Factor? A factor is a number or expression that divides another number or expression without producing a remainder. For example 5 can divide 12 but it will produce a remainder of 2; therefore, 5 is not a factor of 12.

So when we factor a quadratic, why do we call it factoring? Let's take a look at elementary factoring and factoring a quadratic.

$4 \cdot 31 = 124 \;\; so \;\; \frac{124}{4} = 31$ $\qquad\qquad$ $(x+1)(x+4) = x^2 + 5x + 4 \;\; so \;\; \frac{x^2+5x+4}{x+1} = x + 4$

$$\begin{array}{r} 31 \\ 4\,\overline{)124} \\ -12\downarrow \\ o\,4 \\ -\,4 \\ \hline o \end{array} \qquad\qquad \begin{array}{r} x \;\; +4 \\ x+1\,\overline{)\,x^2+5x+4} \\ -x^2-1x \qquad \\ \hline 4x+4 \\ -4x-4 \\ \hline \mathbf{0} \end{array}$$

So why do we factor quadratics and other function? Factoring allows us to determine the x-intercepts of a function since in an x-intercept the $y - value$ is always zero like the remainder. **Make sure to set all equations equal to zero.** There are many different ways to factor. This supportive text explore through examples and will only look at a few different ways to factor depending on the type of function (number of terms in the function).

BINOMIALS – Functions with two terms

$f(x) = -3x - 9 = -3(x+3) \rightarrow so\ when\ does\ x + 3 = 0\ when\ x - 3$ ← factoring from Algebra 1

$g(x) = 4x^5 - 24x^2 = 4x^2(x^3 - 6)$ ←remember to factoring the greatest common divisor between the term

$\rightarrow 4x^3 = 0 \quad and \quad x^3 - 6 = 0$ ← set each factor piece equal to 0 and solve for x

$\rightarrow \;\; x = 0$ and $x = \sqrt[3]{6}$ ← theses represent the x-values of the x-intercept

TRINOMIALS- Functions with three terms

- Factoring a trinomial by ***sight***: $f(x) = x^2 + 2x - 24 \rightarrow$ list the factor pairs of 24

- 24		Sum of Factors
-1	+24	+23
-2	+12	+10
-3	+8	+5
-4	+6	+2

Signs of the numbers can be tricky, make sure that the signs add up to the middle term (-4+6=+2), but will multiply to get the sign on the last term $(- \cdot + = -)$.

Since $-4 + 6 = 2$ make the same value as the middle term we will use these to make the factor pieces $\rightarrow (x-4)(x+6)$ *then set each piece equal to* 0 *and solve for* x,

$$so\ x = 4\ and\ x = -6$$

- Factoring a trinomial using the ***quadratic formula***- this ALWAYS works
  - Formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ where $a \neq 0$.
  - $f(x) = 3x^2 - 5x + 1 \rightarrow a = 3, b = -5, and\ c = 1$ ← Now just plug into formula
  - $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2-4(3)(1)}}{2(3)} = \frac{5 \pm \sqrt{25-12}}{6} = \frac{5 \pm \sqrt{13}}{6}$

- Factoring a trinomial using ***completing the square***- this ALWAYS works but I don't suggest using it for every trinomial
  - This is best illustrated through example: Note leading coefficient or "a" must equal 1

$f(x) = x^2 - 6x + 1$ ← this works great when b is even if not you going to get **fractions**

$x^2 - 6x + 1 = 0$

$x^2 - 6x + \square = -1 + \square$ ← $\square = \left(\frac{b}{2}\right)^2$

$x^2 - 6x + \boxed{9} = -1 + \boxed{9}$

$(x-3)(x-3) = 8$ ← $(x-3)(x-3)$ is done by factoring from sight this step can be skipped

$(x-3)^2 = 8$ ← or skips right to this step $\left(x - \frac{b}{2}\right)^2 = \#$

$x - 3 = \pm\sqrt{8} = \pm 2\sqrt{2}$ ← simplify the radicals

$x = 3 \pm 2\sqrt{2}$

If leading coefficient is a number other than 1, divide all terms by the coefficient *

$$3x^2 - 4x + 1 = 0 \quad so\ \frac{3}{3}x^2 - \frac{4}{3}x + \frac{1}{3} = \frac{0}{3} \quad \rightarrow \quad x^2 - \frac{4}{3}x + \frac{1}{3} = 0$$

This just got really ugly might just want to use quadratic formula instead.

- ***SPECIAL CASES TO KNOW***
  - Difference of Squares: $a^2 - b^2 = (a-b)(a+b)$
    - Example: $9x^2 - 16 \rightarrow a = 3x\ and\ b = 4 \rightarrow so\ (3x-4)(3x+4)$

  - Difference of Cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
  - Sum of Cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
  - Cool Manipulation of making something a quadratic:
    - *Factor* $x^4 - 3x^2 - 4$

      $\rightarrow\ let\ x^2 = u$

      $\rightarrow so\ (x^2)^2 - 3(x^2) - 4\ becomes\ u^2 - 3u - 4$

      $\rightarrow factors\ by\ sight\ to\ (u-4)(u+1)\ which\ means\ (x^2-4)(x^2+1)$

    Now you can set each piece equal to zero and solve