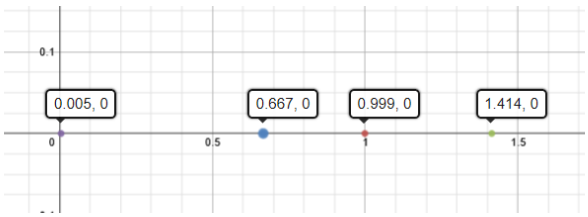
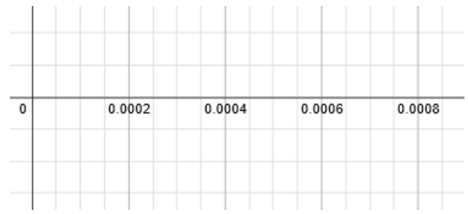
Categories of the Real Numbers

The real numbers are all the numbers that you have seen on the number line and infinitely many more. The real numbers are positive or negative; they can be whole or they can be written by a fraction. Some numbers cannot be written as a fraction, they are just really ugly infinite decimals, like π.





Note: There are infinitely many numbers just between 0 and 1. For example .1, .01, .001, .0001, you can just keep using additional decimal places. The real numbers can be broken into categories. But before we can talk about the categories, we need some key vocabulary. Numbers happen in groups, or sets, and those sets can be broken down further.

Vocabulary:

Set- a collection, or group of elements

Element- a single item in a set

Subset- elements made into a smaller set from a larger set. Subsets belong to the overall larger set.

Empty Set- a set that contains no elements, denoted $\{\emptyset\}$

Union- a set is formed by taking elements from two or more given sets and representing each element uniquely (uniquely → used only once not repeated)

Intersection-a set formed by taking elements from two or more given sets and creating that set by taking elements that are common to the given sets

EXAMPLE:

These are SETS→ $A: \{-1,2,3,5,7\}$ *and* $B: \{-2,0,2,3,5,8\}$

- Set A contains 5 elements and set B contains 6 elements

This is a SUBSET→ $C: \{-1,3,5\}$

- Every element in C is also contained in set A so C is a subset of A

Union of A and B: denoted $A \cup B$→ $\{-2,-1,0,2,3,5,7,8\}$

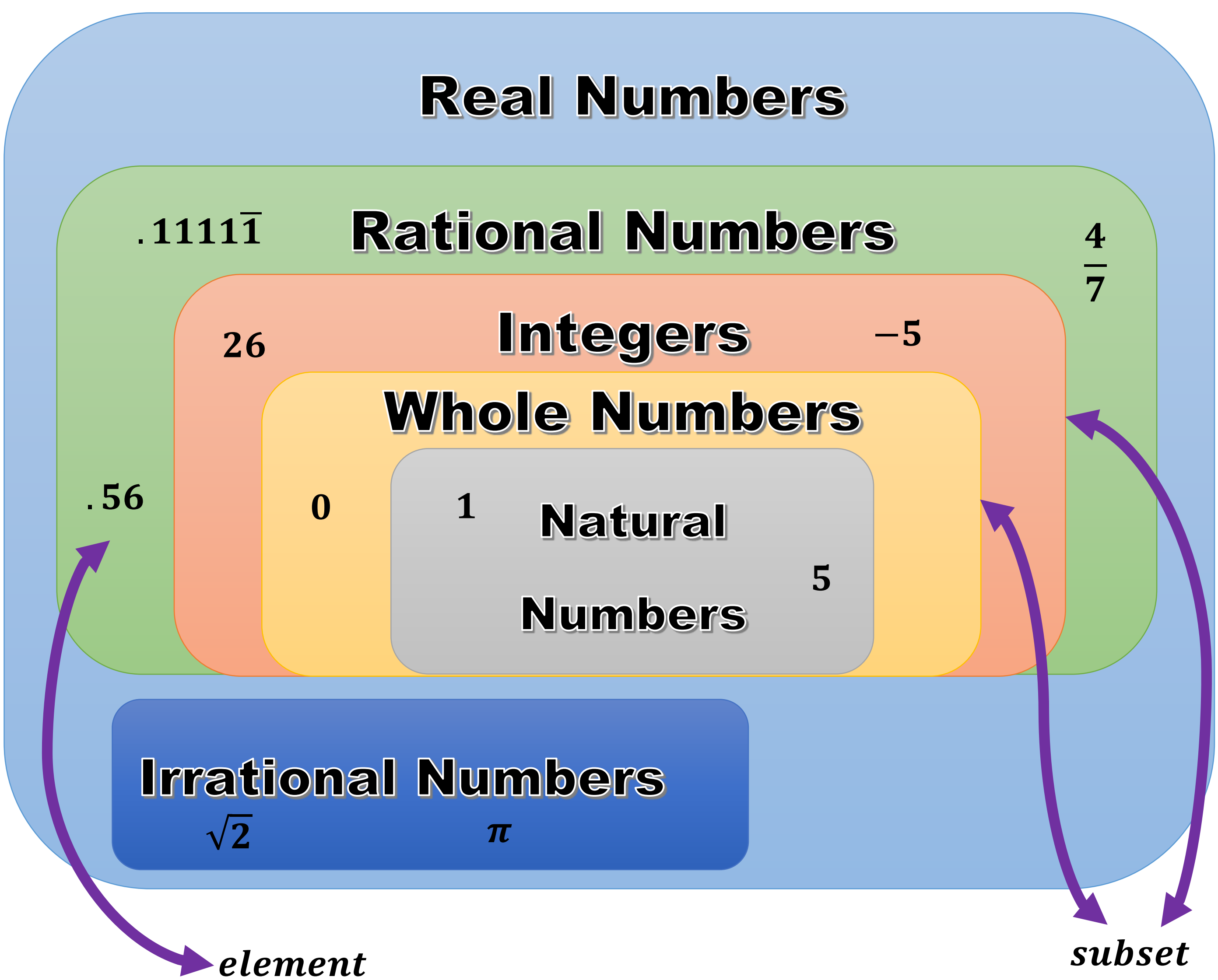
- Notice: 2, 3, and 5 are only represented once in the notation

Intersection of A and B: denoted $A \cap B$→ $\{2,3,5\}$

- Remember what is in common

Categories of the Real Numbers

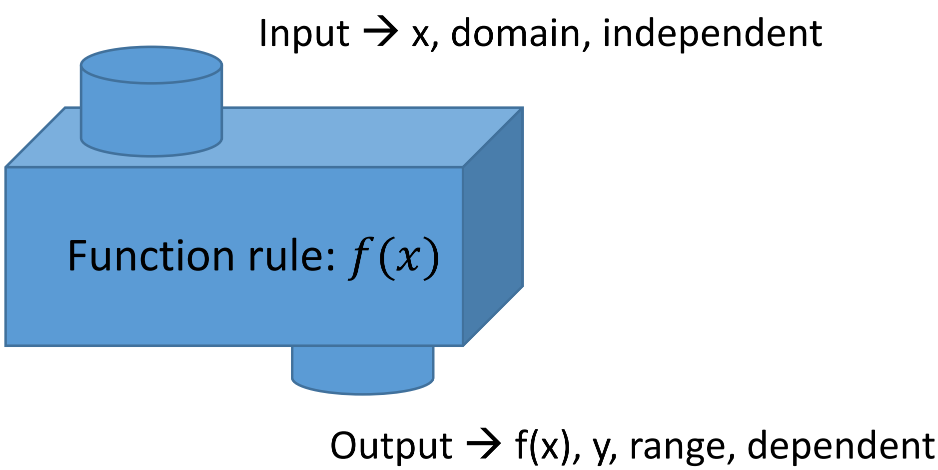
- Real Numbers, symbol $\mathbb{R}$- all numbers found on the number line
  - Fractions, infinite decimals, positive, negative, 0
- Rational Numbers, symbol $\mathbb{Q}$- numbers that can be written as a fraction
  - Remember some decimals can be written as a fraction; decimals that have a repetitive expansion or terminate (or end)
  - *Example*: $.66666\overline{6} = \frac{2}{3}$, $.25 = \frac{1}{4}$
- Irrational Numbers-numbers that cannot be written as a fraction; decimals with infinite expansion without repetition
  - Example: π, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{prime\ number}$, e
- Integers, symbol $\mathbb{Z}$-positive and negative whole numbers and 0
  - This is the typical number line you see in a classroom
- Whole Numbers, symbol $\mathbb{W}$-numbers that do not contain decimal places
  - Example: 0, 1, 2, 3, 4, 5, 6…
- Natural Numbers, symbol $\mathbb{N}$-numbers that we use for counting; NO ZERO
  - Example: 1, 2, 3, 4, 5, 6,…



Function Notation

Relation vs Function: A relation is a set of inputs and output. A function is a type of relation where each input produces a unique output. Here remember the vertical line test that is used for graphs. Function notation is extremely important in higher level math. Reading function notation can cause some trouble at time. For example, write the area, A, of the circle as a function of its radius, r.

KEY: Dependent is a function of the Independent so Area is a function of the radius so A(r). Equation $A(r) = \pi r^2$.



Domain is all values that can be inputted into the function and produce a "DEFINED" output.

$$\text{Example: } f(x) = 3x + 1 \ \ so\ given\ f(2) = 3(2) + 1 = 7$$

$$g(t) = \frac{1}{t} \ \ so\ given\ g(0) = \frac{1}{0} = undefined$$

KEY RULES FOR DOMAIN: Never divide by zero or take the square root of a negative number the results will always be undefined

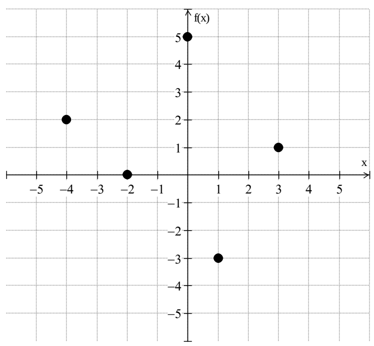
Range is easier to determine if you have a graph of the function. If there is no graph just think about what numbers can be produced →$\frac{1}{x}$ cannot produce zero because of the nature of fractions.

Set Notation-Representing Domain and Range

Set Notation can be represented using a couple different form. To determine which form is best to use you must know if the data is DISCRETE or CONTINUOUS. Discrete is just singular data point that are not connected. Continuous data is infinitely many data points between to endpoint data points.

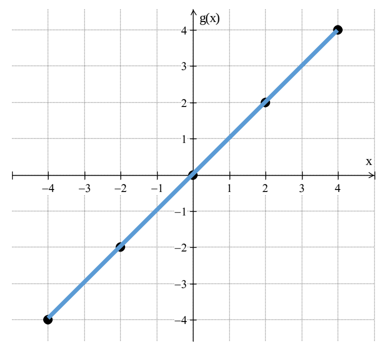
Visual:

$f(x)$



DISCRETE

$g(x)$



CONTINUOUS

TABLES are discrete unless the function is stated to be continuous

Set-Builder Notation uses bracket such as { } to just list discrete data value or will use inequality to represent an infinite set of numbers that fall between two endpoints. Visual will clear this up.

Example using the above graph:

"such that" followed by the restriction for the variable

Domain of $f(x)$: $\{-4, -2, 0, 1, 3\}$

Just a list of single variables

Domain of $g(x)$: $\{x| -4 \leq x \leq 4\}$

Variable being described

Variable

Note that $f(x)$ does not include say the value of 2 because it is not indicated in the list; however, in $g(x)$ the value of 2 being used since it falls between the values of -4 and 4.

Interval Notation-represents a segment of infinite numbers that fall between two endpoint values; cannot represent discrete data values.

Domain of $f(x)$: cannot be represented in Interval notation

Domain of $g(x)$: $[-4\ , 4\ ]$

Important Notation for Set Builder and Interval Notation

Set-Builder:

Brackets: { } start and end of notation

such that → |

< less than

$\leq$ *less than or equal to*

$>$ *greater than*

$\geq$ *greater than or equal to*

Contained in or element of → ϵ

**Infinity symbol is not used in inequalities

Interval Notation

Included value →Square bracket [or]

Not included → Parentheses (or)

infinity uses (or)

Or →∪

Below are the correct ways to write set building notation and interval notation. Let x be the variable followed by the restriction x will follow.

Restriction on X	Set- Builder Notation	Interval Notation
x is all real numbers	$\{x \mid x \in \mathbb{R}\}$	$(-\infty, \infty)$
x is greater than or equal to 0	$\{x \mid x \geq 0\}$	$[0, \infty)$
x is less than 5	$\{x \mid x < 5\}$	$(-\infty, 5)$
x is greater than 3 and x *is less than* or equal to 7 **between statement**	$\{x \mid -3 < x \leq 7\}$	$(-3, 7]$
x less than -1 or x is greater than 4	$\{x \mid x < -1 \text{ or } x > 4\}$	$(-\infty, -1) \cup (4, \infty)$

Basic Rules to follow:

- For a between statement of inequality → only use less than symbols
- Interval Notation is always from least to greatest
- The words "or" and "and" are only used in inequalities; ∪ *and* ∩ are used in interval notation
- Interval Notation indicates a start and stop so it cannot represent discrete data